

DPP4 Set

$$S_{\infty} = \sum_{k=0}^{\infty} \frac{k^2}{4^k}$$

$$\rightarrow S_{\infty} = \frac{1^2}{4} + \frac{2^2}{4^2} + \frac{3^2}{4^3} + \dots + \infty$$

$$S_{\frac{1}{4}} = \frac{1^2}{4^2} + \frac{2^2}{4^3} + \dots + \infty$$

$$\frac{3S_{\infty}}{4} =$$

$$Q2 \quad \begin{array}{c} 2n+1 \quad | \quad 2n+1 \\ \leftarrow \quad \rightarrow \\ \textcircled{3} \quad \textcircled{5} \quad \textcircled{7} \\ \hline 1^3 \quad + \quad 1^3+2^3 \quad + \quad 1^3+2^3+3^3 \quad + \quad \dots \end{array}$$

$$S_n = \sum T_n = \sum \frac{2n+1}{1^3+2^3+\dots+n^3}$$

$$= \sum \frac{(2n+1)4}{(n)^2(n+1)^2}$$

$$= 4 \sum \frac{(n+1)^2 - n^2}{(n)^2(n+1)^2}$$

$$= 4 \sum \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) \text{ Baileh}$$

$$Q3 \quad S_1 = \sum_{k=1}^n k, \quad S_2 = \sum_{k=1}^n k^2, \quad S_3 = \sum_{k=1}^n k^3$$

$$\frac{S_1^4 S_2^2 - S_2^4 S_3^2}{S_1^2 + S_3^2}$$

$$n=1 \rightarrow S_1=1, S_2=1, S_3=1$$

$$\frac{S_2^2 (S_1^4 - S_3^2)}{S_1^2 + S_3^2} = \frac{0}{2} = 0$$

$$Q4 \quad S = \frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \dots +$$

$$\frac{\frac{S}{10} = \frac{1}{10^2} + \frac{2}{10^3} + \dots}{S}$$

$$Q8 \quad S_n = \sum \frac{2r+1}{r^2(r^2+2r+1)}$$

$$\sum \frac{2r+1}{r^2(r+1)^2}$$

$$\sum \frac{1}{r^2} - \frac{1}{(r+1)^2} \text{ Baairien.}$$

$$Q15 \quad \frac{2^4(1^2+2^2+3^2+\dots+n^2)}{(1^2+2^2+3^2+\dots+(2n)^2) - 2(1^2+2^2+\dots+n^2)} > 1.01$$

$$\sum \underbrace{(1 + (n-1)d)}_{\text{AHP}} \cdot r^{n-1}$$

$$d = 5$$

$$r = \frac{1}{5}$$

$$10x - 2 = 0 \Rightarrow x = \frac{1}{5} \quad \left| \quad f\left(\frac{1}{5}\right) = \frac{5}{25} - \frac{2}{5} + \frac{26}{5} = \frac{21}{5} \right.$$

$$=$$

$$f(x) = 5x^2 - 2x + \frac{26}{5}$$

$$f'(x) = 10x - 2 = 0 \Rightarrow x = \frac{1}{5}$$

$$f''(x) = +ve$$

$$f\left(\frac{1}{5}\right) = \frac{5}{25} - \frac{2}{5} + \frac{26}{5}$$

$$= \frac{25}{5} = 5$$

Trigo

$$Q \cos 6^\circ \cos 12^\circ \cos 24^\circ \cos 48^\circ = ?$$

$$Q \cos \frac{\pi}{14} \cos \frac{3\pi}{14} \cos \left(\frac{5\pi}{14}\right)$$

half Angle $\rightarrow$ $\boxed{7\frac{1}{2}^\circ, 22\frac{1}{2}^\circ}$
 $\frac{\pi}{24}, \frac{\pi}{9}$

$$\cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}}, \sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\tan \theta = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$\cot 2\theta = \frac{1 + \cos 2\theta}{\sin 2\theta}$$

AP in Sine Series.

$$1) \sin A + \sin(A+d) + \sin(A+2d) + \dots + \sin(A+(n-1)d)$$

$$= \frac{\sin\left(\frac{n \times \text{diff}}{2}\right)}{\sin\left(\frac{\text{diff}}{2}\right)} \times \sin\left(\frac{1^{\text{st}} + \text{last term}}{2}\right)$$

$$2) \cos A + \cos(A+d) + \cos(A+2d) + \dots + \cos(A+(n-1)d)$$

$$= \frac{\sin\left(\frac{n \times (C.D)}{2}\right)}{\sin\left(\frac{C.D}{2}\right)} \times \cos\left(\frac{1^{\text{st}} \text{ term} + \text{last term}}{2}\right)$$

Q $\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11} = ?$

$$\frac{\sin\left(\frac{5 \times 2\pi}{2 \times 11}\right)}{\sin\left(\frac{2\pi}{2 \times 11}\right)} \times \cos\left(\frac{\frac{\pi}{11} + \frac{9\pi}{11}}{2}\right)$$

$$\frac{\sin\left(\frac{10\pi}{2 \times 11}\right)}{\sin\left(\frac{\pi}{11}\right)} \times \cos\left(\frac{5\pi}{11}\right)$$

$$\frac{2\cos\left(\frac{5\pi}{11}\right) \times \cos\left(\frac{5\pi}{11}\right)}{2\cos\left(\frac{\pi}{11}\right)} = \frac{\cos\left(\frac{10\pi}{11}\right)}{2\cos\left(\frac{\pi}{11}\right)}$$

$$= \frac{\cos\left(\frac{\pi}{11}\right)}{2\cos\left(\frac{\pi}{11}\right)} = \frac{1}{2}$$

Q $\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \sin \frac{3\pi}{n} + \dots + n \text{ terms} = 2 + \sqrt{3}$
 then $n = ?$

$$\frac{\sin\left(\frac{n \times \pi}{2 \times n}\right)}{\sin\left(\frac{\pi}{2n}\right)} \sin\left(\frac{\frac{\pi}{n} + \frac{n\pi}{n}}{2}\right) = 2 + \sqrt{3}$$

$$\frac{\sin\left(\frac{n\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)} \cdot \sin\left(\frac{(n+1)\pi}{2n}\right) = 2 + \sqrt{3} = \boxed{\sin 75^\circ}$$

$$\frac{\sin\left(\frac{\pi}{2} \left(\frac{n+1}{n}\right)\right)}{\sin\left(\frac{\pi}{2n}\right)} = \frac{\sin\left(\frac{\pi}{2} \left(1 + \frac{1}{n}\right)\right)}{\sin\left(\frac{\pi}{2n}\right)} = \frac{\sin\left(\frac{\pi}{2} + \frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)}$$

$$\frac{\cos\left(\frac{\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)} = \cot\left(\frac{\pi}{2n}\right) = \cot 15^\circ$$

$$\frac{\pi}{2n} = \frac{\pi}{12} \Rightarrow \boxed{n=6}$$

Q $S = \sin^2 \theta + \sin^2(2\theta) + \sin^2(3\theta) + \dots + \sin^2(n\theta) = ?$ AP? That But Sym me hai!!!

$$S = \frac{1 - \cos 2\theta}{2} + \frac{1 - \cos 4\theta}{2} + \frac{1 - \cos 6\theta}{2} + \dots + \frac{1 - \cos 2n\theta}{2}$$

$$= \frac{n}{2} - \frac{1}{2} (\cos 2\theta + \cos 4\theta + \cos 6\theta + \dots + \cos 2n\theta)$$

$$= \frac{n}{2} - \frac{1}{2} \frac{\sin\left(\frac{n \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} \cdot \cos\left(\frac{2\theta + 2n\theta}{2}\right)$$

$$= \frac{n}{2} - \frac{1}{2} \frac{\sin(n\theta)}{\sin(\theta)} \cdot \cos((n+1)\theta)$$

Q Find value of

$$S = \sin \theta - \sin 2\theta + \sin 3\theta - \sin 4\theta + \dots + n \text{ terms}$$

$$S = \sin(\pi - \theta) + \sin(2\pi - 2\theta) + \sin(3\pi - 3\theta) + \sin(4\pi - 4\theta) - \dots$$

n terms

$$S = \frac{\sin\left(\frac{n \times (\pi - \theta)}{2}\right)}{\sin\left(\frac{\pi - \theta}{2}\right)} \cdot \sin\left(\frac{(\pi - \theta) + (n\pi - n\theta)}{2}\right)$$

$$= \frac{\sin\left(\frac{n(\pi - \theta)}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \cdot \sin\left(\frac{(n+1)\pi}{2} - \frac{(n+1)\theta}{2}\right)$$

Q $S = \tan \frac{x}{2} \cdot \sec x + \tan \frac{x}{2^2} \cdot \sec \frac{x}{2} + \tan \frac{x}{2^3} \cdot \sec \left(\frac{x}{2^2} \right) + \dots + n \text{ terms} = \tan x - \tan \frac{x}{2^n}$

$$S = \frac{\sin\left(\frac{x}{2}\right)}{\cos \frac{x}{2} \cdot \cos x} + \frac{\sin\left(\frac{x}{2^2}\right)}{\cos \frac{x}{2^2} \cdot \cos \frac{x}{2}} + \frac{\sin\left(\frac{x}{2^3}\right)}{\cos \frac{x}{2^3} \cdot \cos \frac{x}{2^2}} + \dots$$

$$S = \frac{\sin\left(x - \frac{x}{2}\right)}{\cos \frac{x}{2} \cdot \cos x} + \frac{\sin\left(\frac{x}{2} - \frac{x}{2^2}\right)}{\cos \frac{x}{2^2} \cdot \cos \frac{x}{2}} + \frac{\sin\left(\frac{x}{2^2} - \frac{x}{2^3}\right)}{\cos \frac{x}{2^3} \cdot \cos \frac{x}{2^2}} + \dots$$

$$S = \frac{\cancel{\sin x} \cdot \cancel{\cos \frac{x}{2}}}{\cancel{\cos \frac{x}{2}} \cdot \cos x} - \frac{\cancel{\cos x} \cdot \cancel{\sin \frac{x}{2}}}{\cos \frac{x}{2} \cdot \cancel{\cos x}} + \frac{\cancel{\sin \frac{x}{2}} \cdot \cancel{\cos \frac{x}{2^2}}}{\cancel{\cos \frac{x}{2}} \cdot \cancel{\cos \frac{x}{2^2}}} - \frac{\cancel{\cos \frac{x}{2^2}} \cdot \cancel{\sin \frac{x}{2^2}}}{\cancel{\cos \frac{x}{2^2}} \cdot \cancel{\cos \frac{x}{2^3}}} + \frac{\cancel{\sin \frac{x}{2^2}} \cdot \cancel{\cos \frac{x}{2^3}}}{\cos \frac{x}{2^3} \cdot \cancel{\cos \frac{x}{2^2}}} - \frac{\cancel{\cos \frac{x}{2^3}} \cdot \cancel{\sin \frac{x}{2^3}}}{\cancel{\cos \frac{x}{2^3}} \cdot \cancel{\cos \frac{x}{2^4}}} + \dots$$

$$S = \left(\tan x - \tan \frac{x}{2} \right) + \left(\tan \frac{x}{2} - \tan \frac{x}{2^2} \right) + \left(\tan \frac{x}{2^2} - \tan \frac{x}{2^3} \right) + \dots + \left(\tan \frac{x}{2^{n-1}} - \tan \frac{x}{2^n} \right)$$

8 Paper
36 Qs
Tripp Ph.

Trigo Identities with 3 Angles.

$$\tan A \cdot \tan B \cdot \tan C + \tan B \cdot \tan C \cdot \tan D + \tan C \cdot \tan D \cdot \tan E + \tan D \cdot \tan E \cdot \tan A$$

$$1) \sin(A+B+C) = \sin A \cdot \cos B \cdot \cos C + \cos A \cdot \sin B \cdot \cos C + \cos A \cdot \cos B \cdot \sin C$$

$S_1 = \text{Sum of } \tan \theta$

$S_2 = \text{Sum of } \tan^2 \theta$

$S_3 = \text{Sum of } \tan^3 \theta$

$$= \sin A \cdot \cos B \cdot \cos C + \cos A \cdot \sin B \cdot \cos C + \cos A \cdot \cos B \cdot \sin C$$

$$2) \cos(A+B+C) = \cos A \cdot \cos B \cdot \cos C - \sin A \cdot \sin B \cdot \sin C \text{ Now Open.}$$

$$(3) \tan(A+B+C) = \frac{(\tan A + \tan B + \tan C) - (\tan A \cdot \tan B \cdot \tan C)}{1 - (\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A)} = \frac{\sum \tan A - \prod \tan A}{1 - \sum \tan A \cdot \tan B} = \frac{S_1 - S_3}{1 - S_2}$$

$$(4) \tan(A+B+C+D) = \frac{S_1 - S_3}{1 - S_2 + S_4} \quad (5) \tan(A+B+C+D+E) = \frac{S_1 - S_3 + S_5}{1 - S_2 + S_4}$$

$$\tan(A+B+C) = \frac{\sum \tan A - \prod \tan A}{1 - \sum \tan A \cdot \tan B}$$

A) $A+B+C = n\pi$

LHS = $\tan n\pi = 0$

$$0 = \frac{\sum \tan A - \prod \tan A}{1 - \sum \tan A \cdot \tan B}$$

$$\Rightarrow \sum \tan A = \prod \tan A$$

$n=1$

$$\begin{aligned} A+B+C &= \pi \\ \tan A + \tan B + \tan C &= \tan A \cdot \tan B \cdot \tan C \end{aligned}$$

Identity
Proof

$$A+B+C = \frac{\pi}{2}$$

Q If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

then $x+y+z = ?$ $\xrightarrow{x, y, z}$

$$x = \tan A \quad \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$$

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$$

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$$

$$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

$$x + y + z = xyz$$

(B) $A+B+C = (2n+1)\frac{\pi}{2} \Rightarrow$ LHS = $\tan(2n+1)\frac{\pi}{2} = \frac{1}{0}$

RHS = $\frac{\sum \tan A - \prod \tan A}{1 - \sum \tan A \cdot \tan B} = \frac{1}{0} \Rightarrow 1 - \sum \tan A \cdot \tan B = 0$

$$\Rightarrow \sum \tan A \cdot \tan B = 1$$

$$\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1$$

Q If $A+B+C=\pi$ then P.T.

1) $\sum \tan A = \tan A$

$$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

Already Proved

2) $\sum \cot A \cdot \cot B = 1$

Let when $A+B+C=\pi$

$$\frac{\tan A + \tan B + \tan C}{\tan A \cdot \tan B \cdot \tan C} = \frac{\tan A \cdot \tan B \cdot \tan C}{\tan A \cdot \tan B \cdot \tan C} = 1$$

$$\cot B \cdot \cot C + \cot A \cdot \cot C + \cot A \cdot \cot B = 1$$

$$\sum \cot A \cdot \cot B = 1$$

J.I.P

(3) $\sum \tan \frac{A}{2} \cdot \tan \frac{B}{2} = 1$

By Res $A+B+C=\pi$ (given) $\Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

$$\tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \tan \frac{A}{2} = 1$$

$$\sum \tan \frac{A}{2} \cdot \tan \frac{B}{2} = 1 \quad \boxed{J.I.P}$$

(4) $\sum \cot \frac{A}{2} = \pi \cot \frac{A}{2} \Rightarrow$ Q.S is of $\frac{A}{2} \Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$
Wala Result

$$\frac{(\tan \frac{A}{2} \cdot \tan \frac{B}{2}) + (\tan \frac{B}{2} \cdot \tan \frac{C}{2}) + (\tan \frac{C}{2} \cdot \tan \frac{A}{2})}{\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2}} = 1$$

$$\cot \frac{C}{2} + \cot \frac{A}{2} + \cot \frac{B}{2} = \cot \frac{A}{2} \cdot \cot \frac{B}{2} \cdot \cot \frac{C}{2}$$

$$\sum \cot \frac{A}{2} = \pi \cot \frac{A}{2}$$

Q If $x+y+z = xyz$ then P.T.

$$A) \frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \left(\frac{2x}{1-x^2} \right) \left(\frac{2y}{1-y^2} \right) \left(\frac{2z}{1-z^2} \right)$$

$$B) \left(\frac{3x-x^3}{1-3x^2} \right) + \left(\frac{3y-y^3}{1-3y^2} \right) + \left(\frac{3z-z^3}{1-3z^2} \right) = \left(\frac{3x-x^3}{1-3x^2} \right) \left(\frac{3y-y^3}{1-3y^2} \right) \left(\frac{3z-z^3}{1-3z^2} \right)$$

Let $x = \tan A, y = \tan B, z = \tan C$ $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C \Rightarrow \boxed{A+B+C = n\pi}$

$$\Rightarrow 3A + 3B + 3C = 3n\pi \text{ (Bhi Hota)}$$

$$\tan 3A + \tan 3B + \tan 3C = \tan 3A \cdot \tan 3B \cdot \tan 3C$$

$$\left(\frac{3\tan A - \tan^3 A}{1-3\tan^2 A} \right) + \left(\frac{3\tan B - \tan^3 B}{1-3\tan^2 B} \right) + \left(\frac{3\tan C - \tan^3 C}{1-3\tan^2 C} \right) = \left(\frac{3\tan A - \tan^3 A}{1-3\tan^2 A} \right) \left(\frac{3\tan B - \tan^3 B}{1-3\tan^2 B} \right) \left(\frac{3\tan C - \tan^3 C}{1-3\tan^2 C} \right)$$

$$\left(\frac{3x-x^3}{1-3x^2} \right) + \left(\frac{3y-y^3}{1-3y^2} \right) + \left(\frac{3z-z^3}{1-3z^2} \right) = \left(\frac{3x-x^3}{1-3x^2} \right) \left(\frac{3y-y^3}{1-3y^2} \right) \left(\frac{3z-z^3}{1-3z^2} \right)$$

$\tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$ $\leftarrow 2A + 2B + 2C = 2n\pi$ Hoga

$$\frac{2\tan A}{1-\tan^2 A} + \frac{2\tan B}{1-\tan^2 B} + \frac{2\tan C}{1-\tan^2 C} = \left(\frac{2\tan A}{1-\tan^2 A} \right) \left(\frac{2\tan B}{1-\tan^2 B} \right) \left(\frac{2\tan C}{1-\tan^2 C} \right)$$

$$\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \left(\frac{2x}{1-x^2} \right) \left(\frac{2y}{1-y^2} \right) \left(\frac{2z}{1-z^2} \right)$$

Q If $A+B+C=\pi$ then.

1) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

Q ^{LHS} $\sin \rightarrow \text{Prod}$

② $A+B=\pi-C$

$$2 \sin(A+B) \cos(A-B) + 2 \sin C \cos C$$

$$2 \sin(\pi-C) \cos(A-B) + 2 \sin C \cos C$$

$$2 \sin C \cos(A-B) + 2 \sin C \cos C$$

$$\Rightarrow 2 \sin C (\cos(A-B) + \cos(\pi - (A+B)))$$

3) $\sin \rightarrow \text{Prod} \Rightarrow 2 \sin C (\cos(A-B) - \cos(A+B))$

$$\Rightarrow 2 \sin C \left(-2 \sin\left(\frac{A-B+\pi-B}{2}\right) \sin\left(\frac{A-B+A+B}{2}\right) \right)$$

$$= 2 \sin C (+2 \sin(+B) \sin(A))$$

$$\Rightarrow 4 \sin A \sin B \sin C$$

(2) $\cos 2A + \cos 2B + \cos 2C =$

$$2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) + 2 \cos^2 C - 1$$